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BCH-003-001544 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

August – 2021

S - 503 : Statistical

(Statistical Inference)

(Old Course)

Faculty Code : 003

Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Scientific calculator is allowed.

(2) Statistical table will be provided by Institute.

1. **Filling the blanks and short questions. (Each 1 mark)** **(20)**
   1. Estimation is possible only in case of a____________.
   2. A sample constant representing a population parameter is known as ________
   3. If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to___ __
   4. $\sum\frac{X_i}{n}$ for $i = 1, 2, 3, \dots, n$ is a ____________estimator of population mean.
   5. For mean square error to be minimum, bias should be _______.
   6. An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as_____.
   7. If $S = s(X_1, X_2, X_3, \dots, X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for $i = 1, 2, 3, \dots, n$ can be factorised as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, \dots, X_n)$ is a____________.
   8. If a random sample $x_1, x_2, x_3, \dots, x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of σ^2 is _________
   9. For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is________.
   10. Maximum likelihood estimate of the parameter θ of the distribution $f(x,\theta) = \frac{1}{2}e^{-|x-\theta|}$ is _____.
   11. _________is an unbiased estimator of p^2 in Binomial distribution.
   12. The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is __
   13. For a rectangular distribution $\frac{1}{(\beta-\alpha)}$, the maximum likelihood estimates of α and β are **smallest** x and ____________respectively.
   14. If $x_1, x_2, x_3, \dots, x_n$ is a random sample from an infinite population and S^2 is defined as $\frac{\sum(x_i-\bar{x})^2}{n}$, $\frac{n}{n-1}s^2$ is an _________ estimator of population variance σ^2.
   15. Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ______ .
   16. Minimum Chi-square estimators are not necessarily__________.
   17. If a function $f(t)$ of the sufficient statistics $T = t(x_1, x_2, x_3, \dots, x_n)$ is unbiased for $\tau(\theta)$ and is also unique, this is the_____________.
   18. If sufficient estimator exists, it is function of the_____________.
   19. Sample mean is an ____and __________estimate of population mean.
   20. If T_1 and T_2 are two MVU estimator for $T(\theta)$, then __________.

2(a) Write the answer any THREE (Each 2 marks) **(06)**

1. Define Unbiasedness
2. Define Efficiency
3. Define Complete family of distribution
4. Define Uniformly Most Powerful Test (UMP test)
5. Define ASN function of SPRT
6. Find the Cramer Rao lower bound of variance of unbiased estimator of parameter of the probability distribution $f(x,\theta) = \theta e^{-\theta x}$

(b) Write the answer any THREE (Each 3 marks) **(09)**

1. Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.
2. $\frac{\bar{x}}{n}$ is a consistent estimator of p for Binomial distribution.
3. Obtain MVUE of parameter θ for Poisson distribution. Also obtain its variance.
4. Obtain estimator of θ by method of moments in the following distribution
$$f(x;\theta) = \theta e^{-\theta x} \text{ ; where } 0 \le x \le \infty$$
5. Obtain Operating Characteristic (OC) function of SPRT.
6. Give a random sample $x_1, x_2, x_3, \ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta$. Obtain power of the test for testing $H_0: \theta = 1.5$ against $H_1: \theta = 2.5$ where $c = \{x; x \ge 0.8\}$.

(c) Write the answer any TWO (Each 5 marks) **(10)**

1. State Crammer-Rao inequality and prove it.
2. Estimate α and β in the case of Gamma distribution by the method of moments
$$f(x;\alpha,\beta) = \frac{\alpha^\beta}{\Gamma\beta} e^{-\alpha x} x^{\beta-1} \text{ ; } x \ge 0, \alpha \ge 0$$
3. Obtain OC function for SPRT of Binomial distribution for testing $H_0: p = p_0$ against $H_1: p = p_1 (> p_0)$
4. Give a random sample $x_1, x_2, x_3, \ldots, x_n$ from distribution with p.d.f.
$$f(x;\theta) = \theta e^{-\theta x} \text{ ; } 0 \le x \le \infty, \theta > 0$$
Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$.
5. Obtain Likelihood Ration Test:
Let $x_1, x_2, x_3, \ldots, x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 \ne \sigma_0^2$

3(a) Write the answer any THREE (Each 2 marks) **(06)**

1. Define Consistency
2. Define Sufficiency
3. Define Minimum Variance Bound Estimator (MVBE)
4. Define Most Powerful Test (MP test)
5. Obtain likelihood function of Laplace distribution.
6. Obtain an unbiased estimator of θ by for the following distribution
$$f(x;\theta) = \frac{1}{\theta} \text{ ; } 0 \le x < \theta$$

(b) Write the answer any THREE (Each 3 marks) **(09)**

1. Let $x_1, x_2, x_3, \ldots, x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.
2. Obtain an unbiased estimator of population mean of χ^2 distribution.
3. Prove that $E\left(\frac{\partial logL}{\partial\theta}\right)^2 = -E\left(\frac{\partial^2 logL}{\partial\theta^2}\right)$
4. If A is more efficience than B then prove that $Var(A) + Var(B - A) = Var(B)$
5. Use the Neyman Pearson lemma to obtain the best critical region for testing $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.
6. Let p be the probability that coin will fall head in a single toss in order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 head are obtained. Find the probability of type-I error, type-II error and power of test.

(c) Write the answer any TWO (Each 5 marks) **(10)**

1. State Neyman-Pearson lemma and prove it.
2. Obtain MVBE of σ^2 for Normal distribution$(0, \sigma^2)$.
3. If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?
4. For the double Poisson distribution
$$P(X = x) = \frac{1}{2}\frac{e^{-m_1}{m_1}^x}{x!} + \frac{1}{2}\frac{e^{-m_2}{m_2}^x}{x!}; 0,1,2, ...$$
Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$
5. Construct SPRT of Poisson distribution for testing $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.
